

Forces & Moments

Balancing an Airplane

$$F = ma \quad \text{Force} = \text{mass} \times \text{acceleration}$$

$$M = T = rF \quad \text{Moment} = \text{Torque} = \text{distance} \times \text{Force}$$

Seesaw Problem:

- Jack and Jill are on a seesaw. Jack weighs 50 kg and sits 2 m from the **fulcrum** (pivot point). Jane weighs 30 kg. How far away must she sit to balance the seesaw?

Quick Answer:

$$(50 \text{ kg}) \cdot (2 \text{ m}) = (30 \text{ kg}) \cdot (r \text{ m})$$

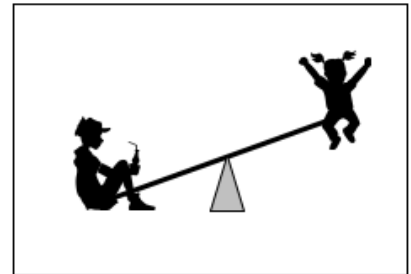
$$r = \frac{50 \cdot 2}{30} \frac{\text{kg} \cdot \text{m}}{\text{kg}} = 3.3 \text{ m}$$

More Complete Approach:

$$M = r \cdot F = 0 \quad \text{for Balance}$$

$$(\text{Force} =) \text{Weight} = \text{Mass} \cdot \text{gravity}; \quad \text{gravity} = 9.8 \text{ m/s}^2 = g$$

$$(r_{\text{Jack}}) \cdot (M_{\text{Jack}}) \cdot g = (r_{\text{Jill}}) \cdot (M_{\text{Jill}}) \cdot g$$



Torque Problem:

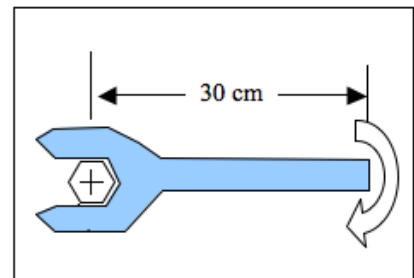
- Sandy has replaced the transmission on her 1983 Escort. The manual says the bolts must be tightened to 3 N•m. Her wrench is 30 cm long. What force should she apply?

$$1 \text{ N (Newton)} = 1 \text{ kg} \cdot \text{m/s}^2$$

$$\text{Equation: } T = r \cdot F$$

$$\text{Knowns: } T = 3 \text{ N} \cdot \text{m}, \quad r = 30 \text{ cm} = 0.3 \text{ m}$$

$$F = \frac{T}{r} = \frac{3}{0.3} \frac{\text{N} \cdot \text{m}}{\text{m}} = 10 \text{ N}$$



How much force should she apply if her wrench is 60 cm?

$$r = 60 \text{ cm} \Rightarrow 5 \text{ N}$$

What happens if she torques the wrench too hard?

If Sandy applies too much force, she's in danger of shearing off the bolt head.

One-sided Seesaw:

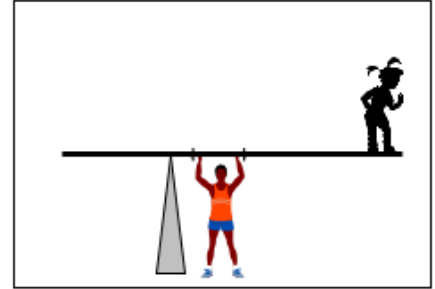
3. Let's look at a different kind of seesaw. Mr. Buff is pushing up 2400 N (≈ 540 lbf) 0.5 m to the right of the fulcrum. Jill (30 kg) is way off to the right. How far away must she be for the seesaw to balance?

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2 \quad g \approx 10 \text{ m/s}^2 \text{ (gravity)}$$

$$M = r \cdot F = 0 \quad \text{for Balance}$$

$$-(r_{\text{Buff}}) \cdot (F_{\text{Buff}}) + (r_{\text{Jill}}) \cdot (W_{\text{Jill}}) = 0$$

$$r_{\text{Jill}} = \frac{(r_{\text{Buff}}) \cdot (F_{\text{Buff}})}{(W_{\text{Jill}})} = \frac{0.5 \cdot 2400 \text{ N} \cdot \text{m}}{30 \cdot 10 \text{ kg} \cdot \text{m/s}^2} = 4 \text{ m}$$

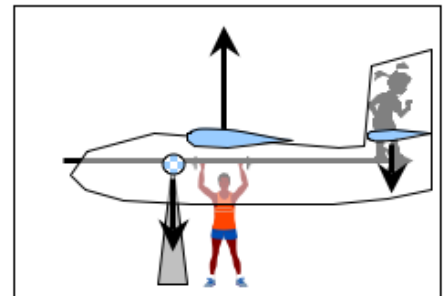


If Jill moves farther away, what will happen to the seesaw?

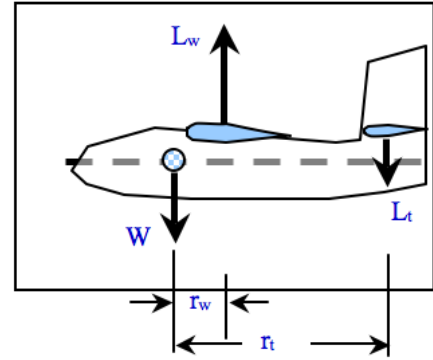
If Jill moves farther away, the see-saw will tilt clockwise.

An airplane is essentially a one-sided seesaw.

4. The pivot point of an airplane is also known as the
 (a) center of gravity (●) (c) rotation point
 (b) balance point (d) fulcrum
The most correct is (a), all would be acceptable.
5. The weight acts through the
 (a) nose of the aircraft (c) center of gravity (●)
 (b) wing center of lift (d) b and c
6. The wing generates
 (a) lift (c) more lift than drag
 (b) lift and drag (d) a, b, and c



7. Refer to the picture on the right. Identify the three **vertical** (up and down) forces acting on the airplane. [Use **W** for the weight arrow, **L_w** for wing lift, and **L_t** for tail lift.] Now, identify the two distances from the **center of gravity** (CG) to the wing center of lift [**r_w**] and to the tail center of lift [**r_t**].



8. What would happen to the nose of the airplane if the only forces were the weight and the wing lift?

The aircraft would pitch nose down.

What keeps this from happening?

The downward lift provided by the horizontal tailplane.

9. The function of the horizontal tailplane is to
- (a) provide lift in the downward direction
 - (b) provide a means for longitudinal stability
 - (c) keep the plane from pitching nose down
 - (d) all of the above

10. For an airplane to be balanced in cruise flight, both the net sum of the forces (ΣF) must be zero and the net sum of the moments (ΣM) must be zero.

- (a) Write the force-balance relationship.

$$\Sigma F = 0 \quad \text{convention: upward force is positive}$$

$$-W + L_w - L_t = 0$$

- (b) Write the moment-balance relationship. (**Hint:** It's easiest about the CG.)

$$\Sigma M = 0 \quad \text{convention: clockwise rotation is positive}$$

$$W \cdot r_{\text{weight}} - L_w \cdot r_w + L_t \cdot r_t = 0$$

but, $r_{\text{weight}} \equiv 0$ (the distance from the weight vector to the CG)

$$-L \cdot r_w + L_t \cdot r_t = 0$$

11. **BONUS:** Why is the CG forward of the wing center of lift?

For Wing Stall – if the wing stalls, the CG will bring the nose down, and the flow will reattach to the upper surface.

12. **The Flaps** are extensions of the main wing near the fuselage (where people & cargo are). They are typically deployed during takeoff and landing. Their function is to increase the wing lift (L_w). Also, because they extend the chord of the wing, the center of lift moves aft (r_w increases).

The Elevator comprises the aft portion of the horizontal tailplane. The pilot deflects the trailing edge of the elevator both up and down by pulling and pushing on the control yoke, respectively. If the pilot pulls the elevator trailing edge up, the tailplane generates more downward lift.



NASA Glenn Icing Research Aircraft
DHC-6 Twin Otter



(a) If flaps are deployed and nothing else changes, what happens to the nose of the aircraft?
It will pitch nose down.

(b) If the pilot doesn't want that to happen, what compensation measures must be taken?
The pilot must generate more downward lift at the tailplane. This is achieved by pulling back on the yoke to bring the elevator trailing edge up.

Now, let's consider a real life problem:

13. In certain weather conditions, ice begins to form on an airplane.

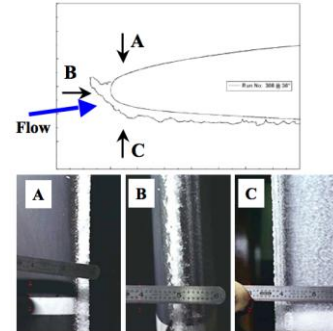
What parts of the airplane are critical to keep ice free?

The wings – to generate *lift*. (Actually any airfoil surface: wing, tailplane, aileron, elevator, rudder, flap).

The propulsor – to generate *thrust*. (For jet engines: the fan blades and engine inlet, for turboprops: the propellers and engine intake, for helicopters: the rotor blades and engine intake)

14. To the right are a cross-sectional view and pictures of an ice-contaminated airfoil.

(Note: a wing, not a tailplane, is shown.)



- (a) What can happen to the pitch control (nose up and down movement) if flaps are deployed and the horizontal tailplane is contaminated with ice?

The aircraft could pitch nose down.

Deploying the flaps increases both L_w and r_w . To maintain level flight, the tailplane must compensate by providing more downward lift. An ice-contaminated tailplane has a reduced maximum lift capacity and a reduced stall margin. If the tailplane is close to the stall point, the downward lift required might exceed its diminished capacity, and the aircraft would pitch nose down.

- (b) How would the pilot use the elevator to compensate?

The pilot would have to pull the elevator trailing edge up.

Topics for advanced discussion:

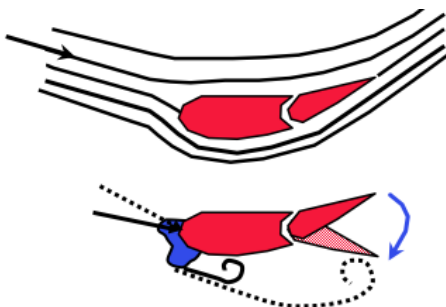
• Tailplane Coefficient of Lift (CL) vs. Tailplane Angle of Attack (AOA).

As the contamination becomes worse, the *maximum* available downward lift decreases (Clean: $CL = -1.25$, Iced: $CL = -0.55$) and the *stall* angle becomes less negative (Clean: $AOA_{stall} = -18^\circ$; Iced: $AOA_{stall} = -8^\circ$).

Note: a tailplane is like an upside-down wing. That's why CL & AOA are negative.

• Flow Separation

The above is due to flow separation on the suction side (lower surface) of the tailplane.



Clean Tailplane:
Attached, "smooth" flow

Iced Tailplane:
Separated flow
(A separation-induced elevator "snatch" is depicted.)

